Section 7-4

Learning objectives:

- Similarity in right triangles

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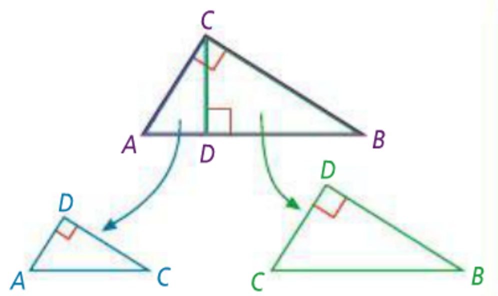
Theorem 7-3

If . . .
$\triangle ABC$ is a right triangle with right $\angle ACB$, and $\overline{CD}$ is the altitude to the hypotenuse



Then . . .
$\triangle ABC \sim \triangle ACD$
$\triangle ABC \sim \triangle CBD$
$\triangle ACD \sim \triangle CBD$

All three triangles are similar to each other!

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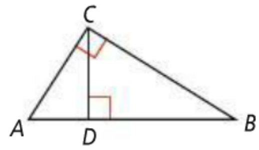
Proof of Theorem 7-3



Given: Right $\triangle ABC$ with right $\angle ACB$ and altitude $\overline{CD}$

Prove: $\triangle ACD \sim \triangle ABC$, $\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle CBD$

Statements	Reasons
1) $\angle ACB$ is a right angle.	1) Given
2) $\overline{CD}$ is an altitude.	2) Given
3) $\overline{CD} \perp \overline{AB}$	3) Definition of altitude
4) $\angle ADC$ and $\angle CDB$ are right angles.	4) Definition of $\perp$
5) $\angle ADC \cong \angle ACB$, $\angle CDB \cong \angle ACB$	5) All right ∡ are $\cong$.
6) $\angle A \cong \angle A$, $\angle B \cong \angle B$	6) Reflexive Property of $\cong$
7) $\triangle ACD \sim \triangle ABC$, $\triangle CBD \sim \triangle ABC$	7) AA ~ Postulate
8) $\angle ACD \cong \angle B$	8) Corresponding ∡ of ~ $\triangle$s are $\cong$.
9) $\angle ADC \cong \angle CDB$	9) All right ∡ are $\cong$.
10) $\triangle ACD \sim \triangle CBD$	10) AA ~ Postulate

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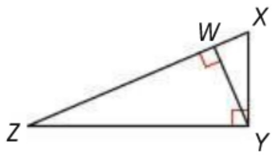
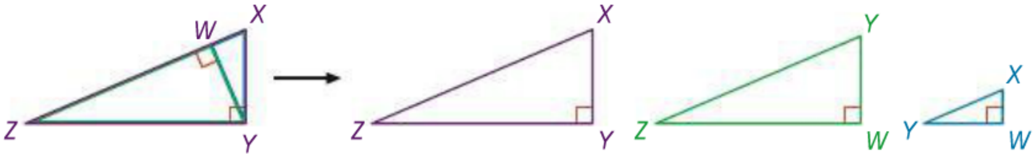
How to use theorem 7-3

What similarity statement can you write relating the three triangles in the diagram?



$\overline{YW}$ is the altitude to the hypotenuse of right $\triangle XYZ$, so you can use Theorem 7-3. There are three similar triangles.



Whenever you have (or can create) a right angle with an altitude from the right angle vertex you can find 3 similar triangles. Remember an altitude meets the opposite side at a right angle!

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The Geometric Mean

Proportions in which the means are equal occur frequently in geometry. For any two positive numbers a and b, the **geometric mean** of a and b is the positive number x such that $\frac{a}{x} = \frac{x}{b}$.

Example: what is the geometric mean of 6 and 15?

Example: what is the geometric mean of 4 and 18?

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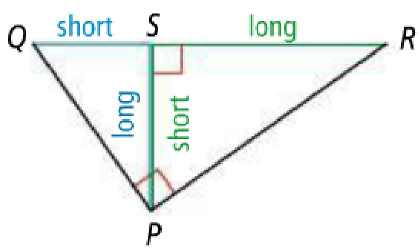
Geometric Mean and Right Triangles



$\triangle SQP \sim \triangle SPR$

$$\frac{\text{short leg}}{\text{short leg}} = \frac{\text{long leg}}{\text{long leg}}$$

$$\frac{SQ}{SP} = \frac{SP}{SR}$$

SP is the geometric mean of *SQ* and *SR*.

Note: $\overline{SP}$ is a shared side.

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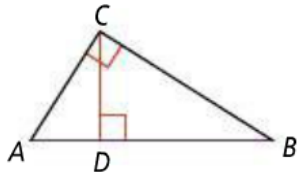
Corollary 1 to theorem 7-3

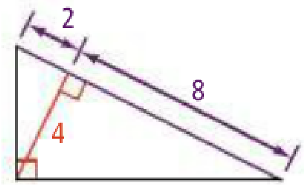
Corollary
The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse.

If . . .



Then . . .

$$\frac{AD}{CD} = \frac{CD}{DB}$$



Segments of hypotenuse → $\frac{2}{4} = \frac{4}{8}$ ← Altitude to hypotenuse

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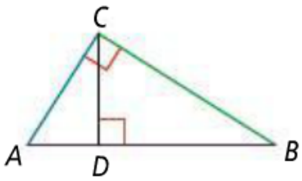
Corollary 2 to theorem 7-3

Corollary
The altitude to the hypotenuse of a right triangle separates the hypotenuse so that the length of each leg of the triangle is the geometric mean of the length of the hypotenuse and the length of the segment of the hypotenuse adjacent to the leg.

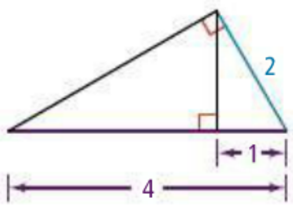
If . . .



Then . . .

$$\frac{AB}{AC} = \frac{AC}{AD}$$

$$\frac{AB}{CB} = \frac{CB}{DB}$$



Hypotenuse → $\frac{4}{2} = \frac{2}{1}$ ← Leg

Segment of hypotenuse adjacent to leg

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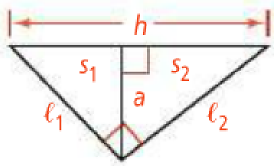
Cheat Sheet

The corollaries to Theorem 7-3 give you ways to write proportions using lengths in right triangles without thinking through the similar triangles. To help remember these corollaries, consider the diagram and these properties.



Corollary 1

$\frac{s_1}{a} = \frac{a}{s_2}$

Corollary 2

$\frac{h}{\ell_1} = \frac{\ell_1}{s_1}, \frac{h}{\ell_2} = \frac{\ell_2}{s_2}$

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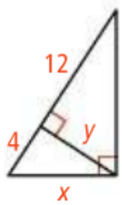
Example

Problem 3 **Using the Corollaries**

Algebra What are the values of x and y?



Use Corollary 2.	$\frac{4+12}{x} = \frac{x}{4}$	Write a proportion.	$\frac{4}{y} = \frac{y}{12}$	Use Corollary 1.
	$x^2 = 64$	Cross Products Property	$y^2 = 48$	
	$x = \sqrt{64}$	Take the positive square root.	$y = \sqrt{48}$	
	$x = 8$	Simplify.	$y = 4\sqrt{3}$	

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